MATH-941 Graph Theory

Credit Hours: 3-0

Prerequisites: None

Course Objectives: Graph theory is a stand-alone branch of pure mathematics that has links across the mathematical spectrum. The primary objective of the course is to introduce students to the beautiful and elegant theory of graphs, focusing primarily on finite graphs.

Previous Knowledge: Basic knowledge of linear algebra is needed.

Core Contents: Basics of graph theory, Path, Cycles, Trees, Matchings, Connectivity and Network Flows, Coloring, Planar graphs.

Detailed Contents: The basics of graph theory: Definition of a graph, graphs as models, matrices, isomorphism, decomposition, paths, cycles, trails, bipartite graphs, Eulerian circuits, vertex degrees and counting, directed graphs.

Trees: Properties of trees, distances in trees and graphs, spanning trees in graphs, decomposition and graceful labeling, minimum spanning trees, shortest paths, trees in computer science.

Matching: Maximum matchings, Hall's matching condition.

Connectivity: Connectivity, edge connectivity, blocks, 2-connected graphs, maximum network flow. Coloring: Vertex coloring, chromatic number, clique number, upper bounds on chromatic number. Planar graphs: Drawing in the plane, dual graphs, Euler's Formula.

Text Book: Douglas B. West, Introduction to Graph Theory, Second Edition, Pearson Education Inc, 2001.

Reference Books:

1. Reinhard Diestel, Graph Theory, Third edition, Springer 2005.
2. J.A. Bondy and U.S.R. Murty, Graph Theory, Springer 2010.
3. B. Bollobas, Modern Graph Theory, Springer 1998.

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)

Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown	
Week	**Topics**
1	Definition of graphs: loops, multiple edges, simple graphs, neighbors. Graph as models:Complement, clique, independent set, bipartite graphs
2	Chromatic number, k-partite graphs, path, cycle, subgraphs. Matrices and Isomorphism:adjacency matrix, incidence matrix, degree of vertex
3	Isomorphism, n-cycle, complete graph, complete bipartite graphs. Decomposition: self-complementary graphs, decomposition
4	Triangle, paw, claw, kite, Petersen graph, girth. Connection in graphs: walks, trail, u,v-walkand path, internal vertices, length of walk and path.
5	Connected and disconnected graphs, components of graph, isolated vertex, cut-edge, cut- vertex, induced subgraphs, union of graphs, Eulerian graphs, Eulerian circuits, even graph
6	Vertex degrees and counting: degree of vertex, regular and k-regular graphs, neighborhood, order of a graph, Counting and bijections: degree sum formula, k-dimensional cube. Graphicsequence, introduction of directed graphs
7	Trees: acyclic graph, forest, leaf, spanning subgraphs, spanning trees, star, properties oftrees
8	Distances in trees and graphs: distance, diameter, eccentricity, radius, center of a graph,Wiener index, contraction of edges, graceful labelling
9	**Mid Semester Exam**
10	Minimum spanning tree: Kruskal Algorithm, Shortest path: Dijkstra's Algorithm
11	Trees in Computer Science: Rooted tree, children, ancestors, descendants, rooted plane tree,binary tree, left child, right child
12	Matchings: matching, perfect matchings, maximum and maximal matchings, M-alternatingand augmenting paths, symmetric difference, Hall's matching

	condition
13	Connectivity: vertex cut, connectivity and k-connected graphs, edge-connectivity, edge-connectivity and k-edge-connected graphs,
14	Network Flow Problems: Network, capacity, source and sink vertex, flow, maximum network flow, Ford-Fulkerson labeling algorithm
15	Coloring of graphs: k-coloring, proper coloring, k-colorable graphs, chromatic number, k-chromatic graphs, greedy coloring algorithm
16	Planar graphs: curve, polygonal curve, crossing, planar graphs, planner embedding, closed curve, simple curve, region, faces, dual graphs, Euler's formula
17	Review
18	**End Semester Exam**